J. Opt. Commun., Vol. 32 (2011), pp. 201–204 DOI 10.1515/JOC.2011.037

Minimization of Point Light Source Coordinates Determination Error on Photo Detectors

Ivan Kruzhilov[1,*]

[1] Department of Applied Mathematics, Moscow Power Engineering Institute (Technical University), Moscow, Russia

Abstract. In this article it is examined the influence between the projection size of the point light source and the detection error of coordinate center when projecting on the photo-sensitive matrix. With the help of simulation it is formed the dependence of detection error of coordinate from magnitude of dispersion radius projection in case of different position of light source relative to the receiver. Analized operating characteristic of two delimitation of coordinate algorithms: using maximal likehood method and using the centroid method. It is find of the ratio between the dispersion radius and size of receiver in the case of minimal error.

Keywords. Point light source, detection error of coordinate, astronavigation, star tracker, centroid, maximal likehood method.

PACS®(2010). 02.50.Ey.

1 Introduction

Coordinate detection of the point light source (PLS) is one of the main problem of object position location in the space. For analysis of the light source location it is widely used matrix photosensitive receivers like for example charge-coupled device (CCD) matrix or Active Pixel Sensors (APS).

Light from the PLS after going through the optical system is projected to an analysis surface plane as a unsharp spot. Size and projection form depends on optical system. Coordinate error detection of PLS depends on matrix properties, parameters of projected image and on algorithm used for coordinate check. Parameters of light signal projections exerting the main influence on accuracy of coordinate check are the next: signal power, distribution function of signal's energy, picture projection size, signal accumulation time, position of the PLS relative to the photoreceiver and interference intensity.

In general case, endless reducing of dispersion radius has no leading effect of increasing positioning accuracy. Dispersion radius with which a minimal error of evaluation is reached will be called optimal radius.

This work have for an object to examine the hypothesis about the optimal projection size existence. For proving this hypothesis and finding out the optimal projection size relative to the size of discretization element it was used imitating simulation.

Reviewing problem is of great applied importance. Examining stars as PLS it is possible to get the results, which can be used for the stars orientation optoelectronic devices perfection. Increasing accuracy of star position detection on the each frame enables to decrease total time of satellite orientation.

2 Review

Coordinates estimations methods can be splited in two classes: those who demand apriory information about analyzed signals and noises, and those, who don't demand that kind of information [1]. Centroid methods and methods based on Fourier transformation don't use any additional information about signal form and interference power. Fitting method only use information about signal form, when maximum-likelihood method also need information about signal and interference power. For the fitting method all kinds of parameters must be estimated (for example, the Gauss form of signal needed four parameters). In his article [2] Mighell et al. suggested Levenberg method for the signal parameters estimation. Simulation data from the article [1] show, that errors in centroid methods greatly overcome errors of fitting method and Fourier transformation. The advantage of centroid method in comparing with Fourier transformation based on simplicity of it implementation. That's why centroid method had proof himself highly dependable and often seen in star trackers [3–5].

The centroid algorithm optimal radius research was make in [6]. Sadly, authors not specified such important parameter of simulation as position the PLS relative to the photoreceiver. Simulation runs for stars 9 m and weaker. On simulation parameters we used, integration time was 30 ms, readout noise 30 electrons and pixel size 1 arcsec-

* **Corresponding author:** Ivan Kruzhilov, Department of Applied Mathematics, Moscow Power Engineering Institute (Technical University), Krasnokasarmennaya-19-30, Moscow, 111116, Russia; E-mail: krujeelov@mail.ru.

Received: March 27, 2011. Accepted: September 23, 2011.

onds, optimal radius equal approximately 1.5–2 RMS. Because of weak stellar brightness a size of analysed field was 2–4 pixels.

In the works [7, 8] was estimated error's lower limit of PLS coordinates determination based on Rao–Cramer inequality. According to the works [7, 8] best value ratio of scattering radius to pixel size consist 0.3–0.4. However, Rao–Cramer inequality points lower boundary of possible coordinate's estimation error. It is not known, is there a method, that reaches this lower boundary. If those method exist, it will be maximum-likelihood method. In this work maximum-likelihood method was chosen for optimal radius determination problems research, because among the other methods his variance more close to the errors lower boundary than any other method. Finally a two methods was chose for the research – centroid and maximum-likelihood method.

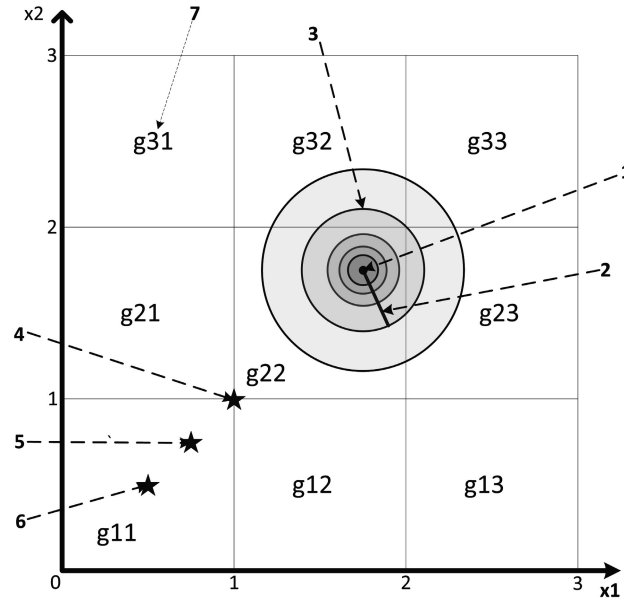

Figure 1. Projection range of picture G, 1 – center of the PLC projection $\mathbf{x_0}$, 2 – dispersion radius r, 3 – equipotential line of signal power, 4, 5, 6 – center location of the PLC projection $\mathbf{x_0}$ relative to model receiver, 7 – subfield g of the sub range G equal to elementary photoreceiver.

3 Methodology and Simulation Results

Model takes as a principle that photosensitive matrix consists of the pixel aggregate adjoined to each other without spacing between them. Geometrically the matrix planes are the field of image G projection (Figure 1). Range G is divided to rectangular adjoined non-intersecting sets g_{ij} conformed to pixels of the matrix: $G = \bigcup_{i,j} g_{i,j}$, $\bigcap_{i,j} g_{i,j} = \emptyset$.

Energy accumulated by each pixel is measured by quantity of electrons. Electrons in their turn are converted to kvants and processed with the help of analog-digital converter (ADC).

The model is founded on the hypothesis that quantity of quantum ADC from each elementary photo receiver (i, j) is random quantity N_{ij} which may be described by Poisson distribution. That is $N_{ij} \sim \mathrm{Po}(L_{ij} \cdot t)$. Where L_{ij} is intensity of light stream falling on pixel (i, j) which is equal to:

$$L_{i,j} = \int_{gij} L(\mathbf{x})\, d\mathbf{x} \tag{1}$$

From the expression (1) $L(\mathbf{x}) = L_1(\mathbf{x}) + l_0$-two-dimensional distribution light intensity function, $\mathbf{x} = (x_1, x_2)$-coordinates on the receiving matrix, l_0-intensity of interferences applied equally to unit of area. $L_1(\mathbf{x})$-light energy density from the useful signal equal to

$$L_1(\mathbf{x}) = L_1 \cdot f(\mathbf{x} - \mathbf{x_0}). \tag{2}$$

In the expression (2) function $f(\mathbf{x})$ means spatial type of power energy signal distribution on the projection. $\mathbf{x_0} = (x_{01}, x_{02})$-coordinates of PLS projection for rays passing the ideal optical system. It is supposed that light power in the range G from the light source L_1 does not depend on PLS location.

$$L_1 = \int_G L_1(\mathbf{x}) d\mathbf{x} = \text{const.}$$

For the researching function $f(\mathbf{x})$ was assumed as:

$$f(\mathbf{x}) = f(x_1, x_2) = \exp(-(x_1^2 + x_2^2)/2r^2)/(2\pi r^2), \tag{3}$$

where r is the dispersion radius.

Problem of center projection coordinate check PLC contains the following:

It is required on the totality of date N to estimate parameter $\mathbf{x_0}$, considering as known parameter l_0 and L_1. By accumulating imitation of the charges for the each matrix pixel it was calculated, according to (1), the intensity falling on the pixel light stream.

We get as the result the intensity of falling stream L_{ij} on the step of each calculating process for signal accumulation time was generated random matrix N. According the each random matrix N was estimated the center projection $\mathbf{x_0}$ coordinate. After carrying out experiment series estimated values of $\mathbf{x_0}$ were compared with true value (which was used for N matrix generation). According to the results of this comparisons were get the static characteristics of coordinate check error.

For the definition location was used maximum-likelihood method (MLM) and centroid estimation –

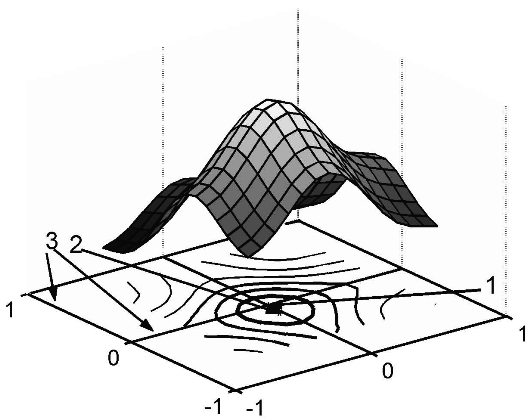

Figure 2. Implementation of likelihood function. Projection center is situated in the pixel's adherent point. 1 – True coordinate of center projection, 2 – coordinate of center projection according to MLM, 3 – domain boundary of pixel.

weighed mean method. The ideas of these algorithms differ from each other. Use MLM needs primary information about form and power of signal and interferences. Centroid algorithm on the contrary does not require any complementary information.

Used in the work estimate is based on calculating of likelihood function maximum. As far as random quantity N_{ij} is independent, likelihood function look like that:

$$P(N|\mathbf{x_0}) = \prod_{i=i_0-kr}^{i_0+kr} \prod_{j_0-kr}^{j_0+kr} \frac{(L_{ij}t)^{Nij}}{(N_{ij})!} \exp(-L_{ij}t).$$

Logarithm of likelihood function for parameter $\mathbf{x_0}$ estimating is equal to:

$$P(N|\mathbf{x_0}) = \sum_{i=i_0-kr}^{i0+kr} \sum_{j_0-kr}^{j_0+kr} \left(-t \int_{g_{ij}} L(\mathbf{x}) d\mathbf{x} + N_{ij} \ln \left(t \int_{g_{ij}} L(\mathbf{x}) d\mathbf{x} \right) - \ln(N_{ij}!) \right). \quad (4)$$

Right-hand member (4) does not depend evidently on x_0 but it should be mentioned that according to (2) function $L(x)$ depends on estimated parameter. x_0 is thought to be the maximum of function (4) which is represented on Figure 2. It is impossible to find analytically a point, where function (4) is minimal.

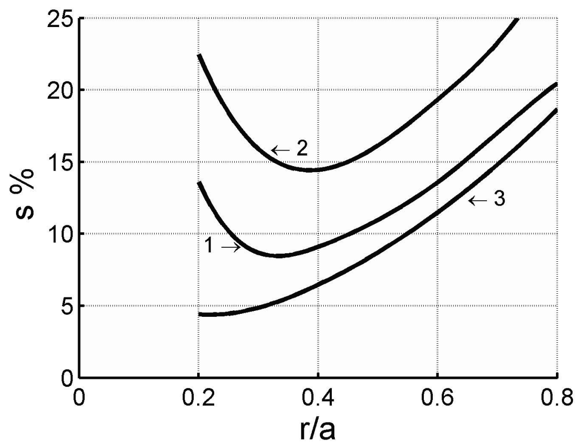

Figure 3. Dependence root-mean-square error of coordinate location on dispersion radius (SC).

If signal form is Gaussian and pixel has square form size a then integral take on following form:

$$\int_{g_{ij}} L(\mathbf{x}|\mathbf{x_0}) d\mathbf{x} = \frac{L_1}{2\pi r^2} \int_{ia}^{ia+a} \times \int_{ja}^{ja+a} \exp\left(-\frac{(x_1 - x_{01})^2 + (x_2 - x_{02})^2}{2r^2} \right) dx_2 dx_1,$$

$$\int_{g_{ij}} L(\mathbf{x}|\mathbf{x_0}) d\mathbf{x} = L_1 \times \left[\Phi\left(\frac{ia + a - x_{01}}{r} \right) - \Phi\left(\frac{ia - x_{01}}{r} \right) \right] \times \left[\Phi\left(\frac{ja + a - x_{02}}{r} \right) - \Phi\left(\frac{ja - x_{02}}{r} \right) \right],$$

where Φ is the Laplace function. That representation of the likelihood function make a running of computing experiment a lot easier.

Was researched characteristics of algorithms for the radius r various from 0.2 to 0.8 dimension of discretization element with incremental step 0.1. There were observed three different cases of center projection location (Figure 1): in the center of pixel-point 6, boundary-point 4 and the point 5 situated in the center of segment between two previous points. On an average point 5 as the projection center gives more information about characteristics of algorithms.

Rao–Cramer boundary evaluation were made in following articles [7, 8]. According to [7] dispersion error of coordinate center projection $x_{01}(x_{02})$ using the MLM is:

$$D_{x_{01}} = \left\{ \frac{Ll^2 t}{l_0} \sum_{i,j} \left[\int_{g_{ij}} \frac{df(\mathbf{x} - \mathbf{x_0})}{dx_{01}} d\mathbf{x} \right]^2 \right\}^{-1}. \quad (5)$$

Results of modeling are represented on the Figures 3–5, where are the graphs of dependence root-mean-square error of center coordinate location (in % from demention of

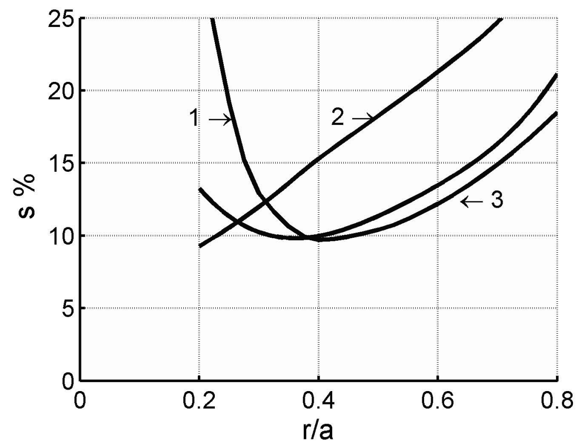

Figure 4. Dependence root-mean-square error of coordinate location on dispersion radius.

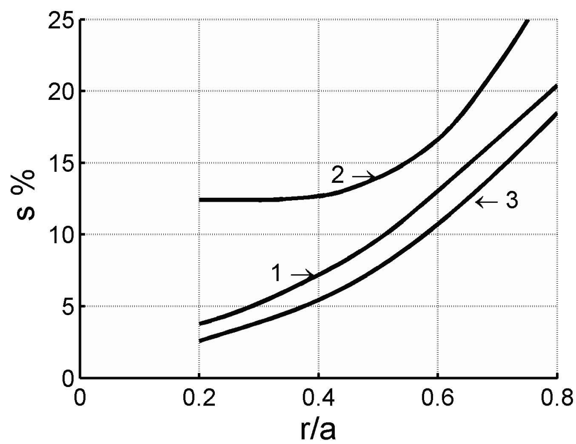

Figure 5. Dependence root-mean-square error of coordinate location on dispersion radius (SC).

pixel) on relative size r. Graphs shows three characteristics: root-mean-square error of MLM, root-mean-square error of centroid algorithm and estimation (5).

Apparently from graphs the error of coordinate location from SC depends a lot on both center projection location and applied algorithm. Generally MLM gives the lesser error than centroid. On an average error of coordinate location got from centroid exceeds MLM error almost twice as much.

RMS of centroid method equal approximately $20 \pm 5\%$. This value is approximately equal ratio of pixel's size to signal-to-noise ratio (SNR) according to evaluation from [4]. For general case simulation results are close to evaluation from [7]. However, Rao–Cramers boundary distinguish from simulation results with little scattering radius. In that case Rao–Cramers boundary is close to MLM evaluation only asymptotically.

4 Conclusion

According to the results was turned out the optimal dispersion radius for PLC center coordinate location is equal to 0.3–0.35 from elementary photoreceiver.

On the basis of the results it can be concluded that putting into practice the dispersion radius of the PLS projection in 1.5–2 less again as it is used today in optoelectronic star navigation devices makes it possible to reduce the error by 15–20%. For the real devices the decreasing of error of center coordinate location means decreasing of the detecting time of coordinate location of satellite in the space at the expense of decreasing of the selection interferences like "luminous point" and getting more exact information about interstellar angles.

References

[1] Canabal H., Alonso J., Bernabeu E., Laser beam deflectometry based on a subpixel resolution algorithm, Optical Engineering, Vol. 40(11) (2001), pp. 2517–2523.

[2] Mighell, K., Algorithm for CCD Stellar Photometry Astronomical Data Analysis, Mighell, K., Astronomical ASP Conf. Ser. Data Analysis Software and Systems VIII, Vol. 172 (1999), pp. 317–328.

[3] Srivastava, M. K., Prabhudesai, S. M., Tandon, S. N., Studying the Imaging Characteristics of Ultra Violet Imaging Telescope (UVIT) through Numerical Simulations, Publications of the Astronomical Society of the Pacific, Vol. 121 (2009), pp. 621–633.

[4] Avanesov G. A., Ziman J. L., Krasikov V. A., Definition Algorithms of a Space Vehicle Orientation According to Airborne Astronomical Measurements, Priborostroenie, Vol. 46(4) (2003), pp. 31–37.

[5] Liu, H.-B., Tan, J.-C., Huang, S.-H., Method for decrease of the centroid error of star image caused by stellar spectrum, Proceedings of the SPIE, Vol. 7511 (2009), pp. 75110U–75110U-6.

[6] Secroun A., Lampton, M. , Levi, M., A high-accuracy, small field of view star guider with application to snap, Experimental Astronomy, Vol.12(2) (2001), pp. 69–85.

[7] Vedoseev V. I., Analysis of optical signal in matrix photodetector, Priborostroenie, Vol. XXVII(7) (1984), pp. 70–78.

[8] Chena, H., Rao, C. Accuracy analysis on centroid estimation algorithm limited by photon noise for point object, Optics Communications, Vol. 282(8) (2009), pp. 1526–1530.